STAT 2593

Lecture 003 - Measures of Location

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Measures of Location

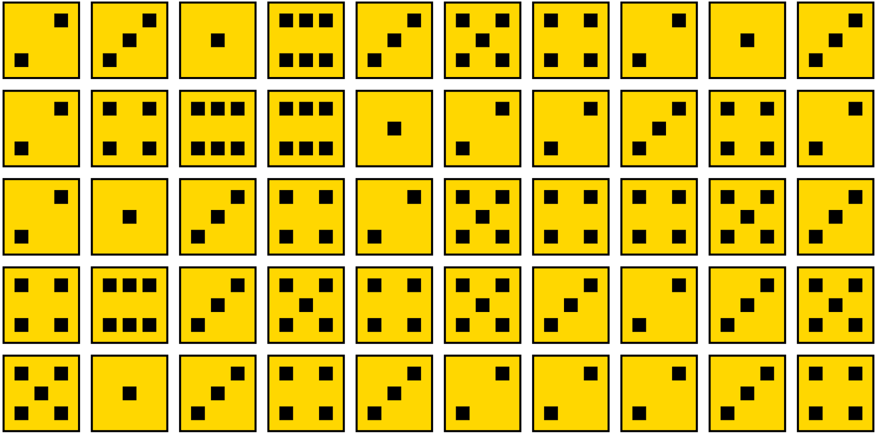
Learning Objectives

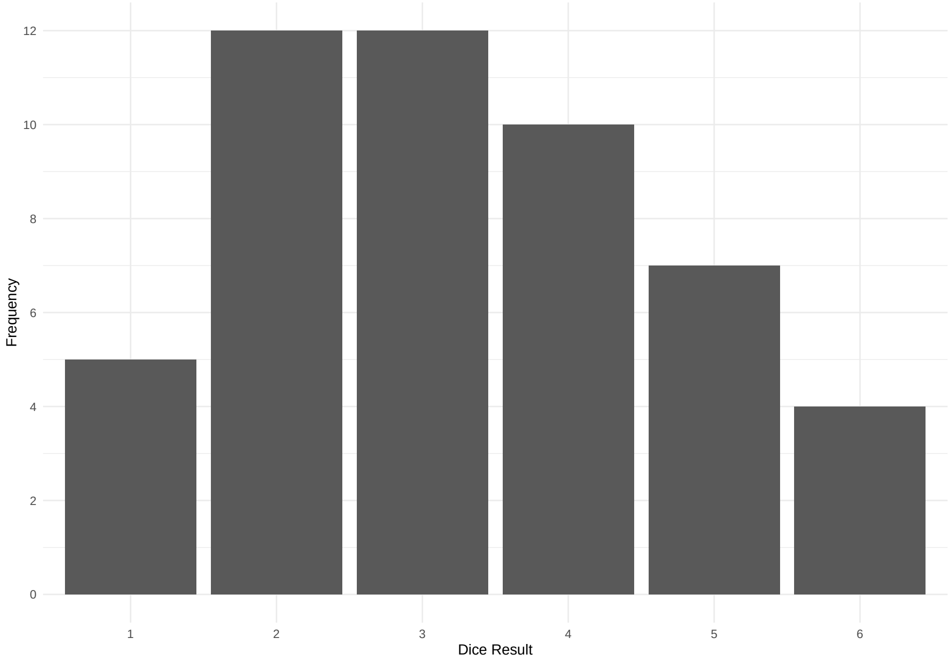
1. Understand and interpret the mean, median, and mode
2. Understand and interpret the sample proportion

Where is our Data?

Given a large dataset, how do we understand where observations typically fall?

Success: 50 of 50 (100%)




12
10
8
6
4
2
0
Frequency
1
2
3
4
5
6
Dice Result

Measures of Location

- **Sample mean** is the standard average of a distribution.

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$$\text{median} = \begin{cases} \left(\frac{n+1}{2}\right)^{\text{th}} \text{observation} & n \text{ is odd.} \\ \text{Mean of } \left(\frac{n}{2}\right)^{\text{th}} \text{ and } \left(\frac{n}{2}+1\right)^{\text{th}} \text{ observations} & n \text{ is even} \end{cases}$$

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- **Sample mode** is the most common (set of) observation(s).

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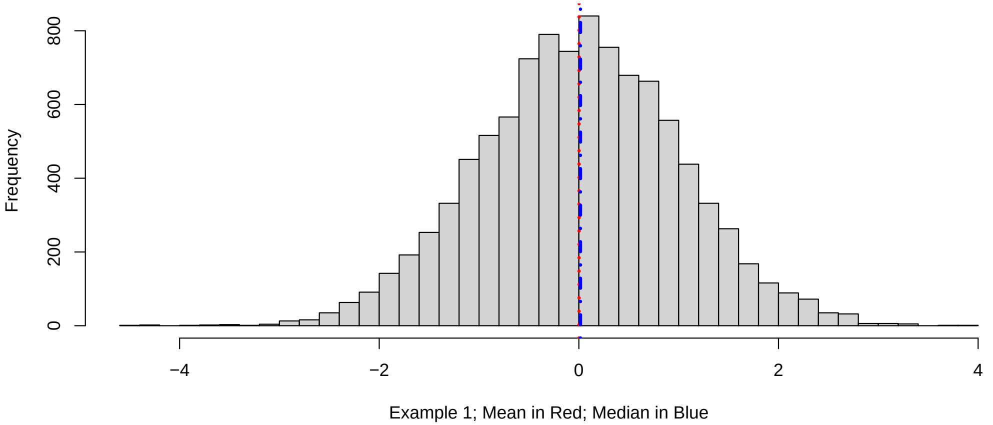
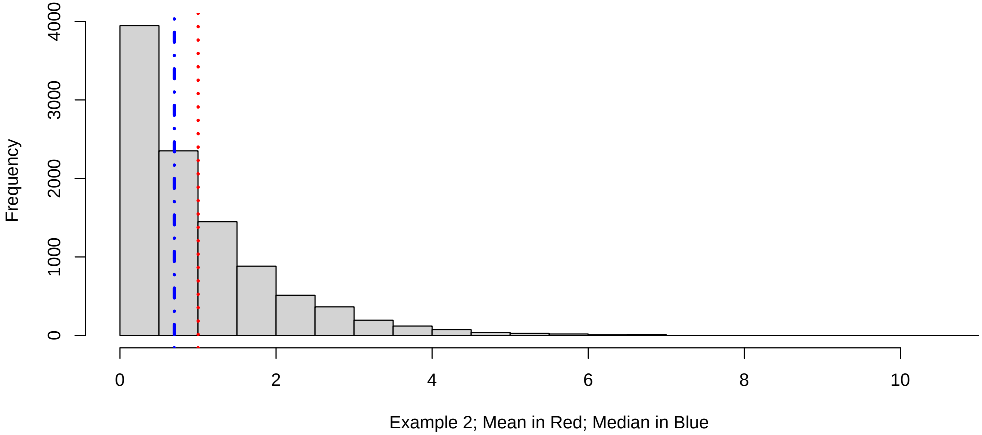
- If data are skewed, the mean is *pulled* towards the long tail of the distribution.
  - In this way, the mean is more sensitive to *skewed* outliers than the median.
- We generally prefer the median if data are skewed, and the mean otherwise.

Examples

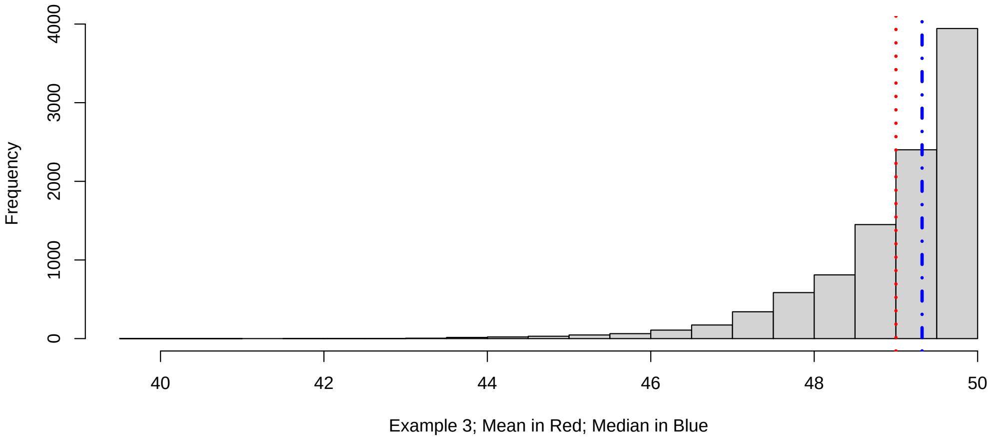


Frequency
0
200
400
600
800
−4
−2
0
2
4
Example 1; Mean in Red; Median in Blue

Examples

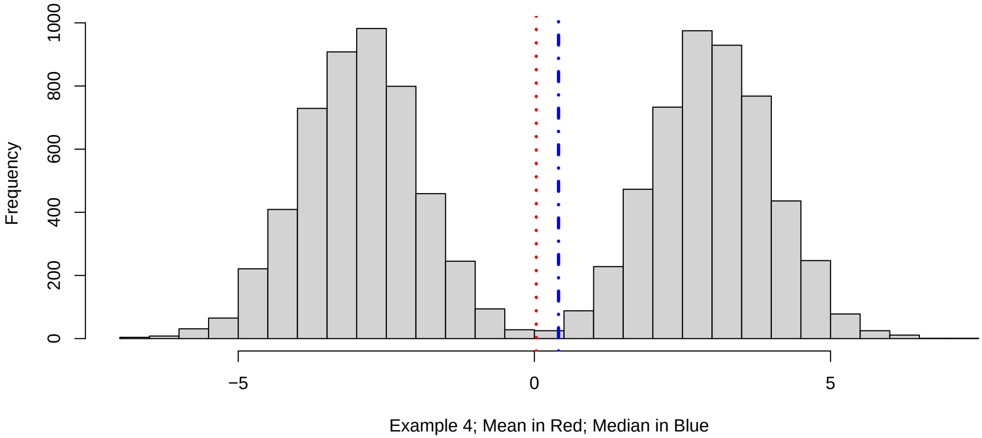

Frequency
0
1000
2000
3000
4000
0
2
4
6
8
10
Example 2; Mean in Red; Median in Blue

Examples



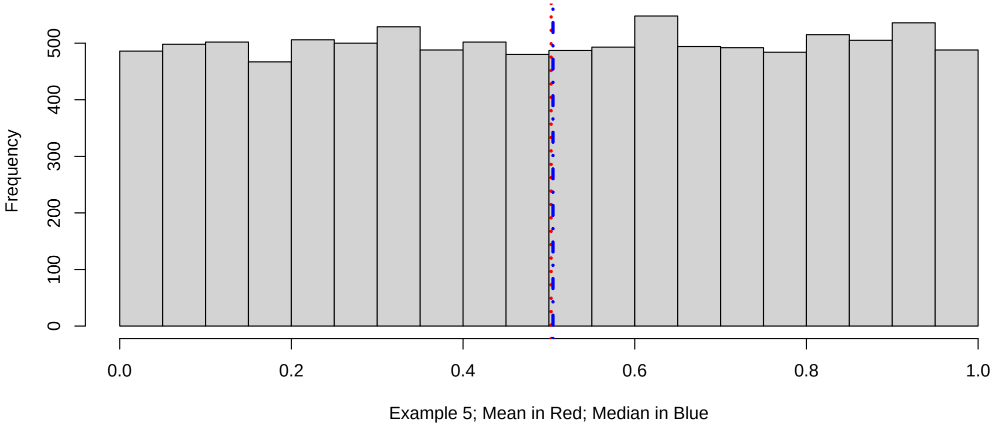
Frequency
0
1000
2000
3000
4000
40
42
44
46
48
50
Example 3; Mean in Red; Median in Blue

Examples

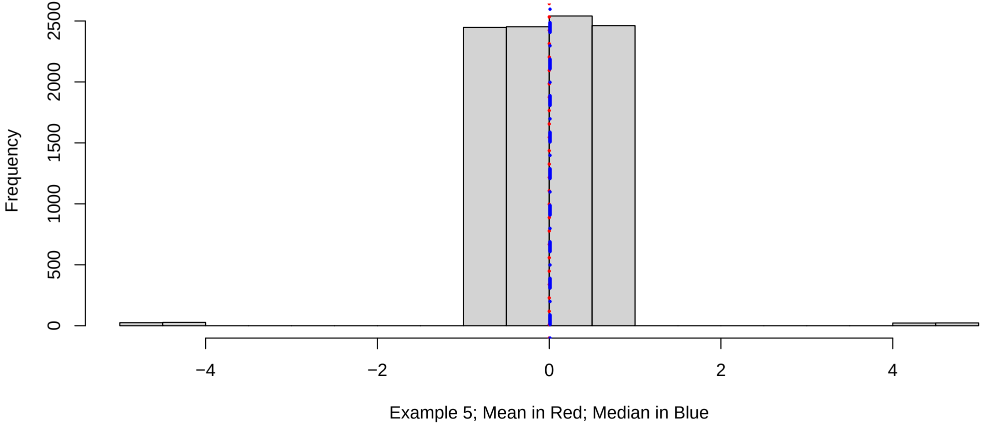


Frequency
0
200
400
600
800
1000
−5
0
5
Example 4; Mean in Red; Median in Blue

Examples


Frequency
0
100
200
300
400
500
0.0
0.2
0.4
0.6
0.8
1.0
Example 5; Mean in Red; Median in Blue

Examples



Frequency
0
500
1000
1500
2000
2500
−4
−2
0
2
4
Example 5; Mean in Red; Median in Blue

Sample Proportion

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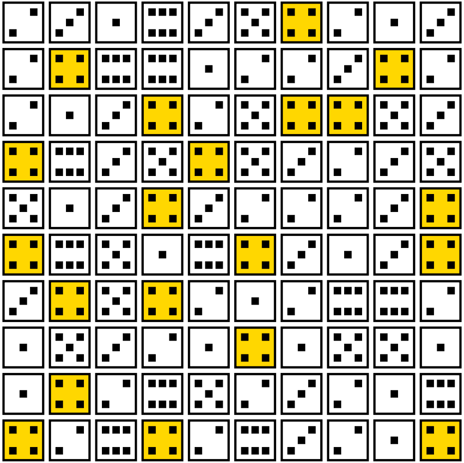
- Define $z_{i,j} = I(x_i = c_j)$, where $I(\cdot)$ is an indicator function.
- Then, we can write the j-th sample proportion as

$$p_j = \bar{z}_{\cdot,j} = \frac{1}{n}\sum_{i=1}^{n} z_{i,j}.$$

Example

Success: 20 of 100 (20%)



Summary

- Measures of location indicate what *usually* happens in a dataset
- The mean, median, and mode can be computed for quantitative variables
- The mean and median are most commonly used; the median is preferable for skewed data
- The sample proportion is used for indicating the location of a categorical variable